The Role of Alternative Assets in Portfolio Construction

John M. Mulvey
Princeton University
mulvey@princeton.edu

Woo Chang Kim
Princeton University
wookim@princeton.edu

May 20, 2007

Abstract

This article discusses the role of non-traditional assets in constructing a portfolio for long-term investors. Alternative investments can provide novel patterns of returns and increase diversification benefits. In turn, investors can apply leverage in conjunction with wide diversification to improve their risk-adjusted performance. The concepts are illustrated via popular categories of alternative investments. We focus on long-term investors who are able to rebalance their portfolios on a regular basis. Careful risk management is critical in this domain.

1. Motivation

In this paper, we discuss the role of alternative investments within the context of asset allocation for long-term investors. We define alternative assets as the most popular private securities/contracts -- hedge funds; managed futures (commodities, currency, and fixed-income); and private equity (venture capital and leveraged buyouts). These asset categories cover a wide assortment of investment strategies [1, 2, 3, and 4]. For example, there are over a dozen subcategories of hedge funds. Unfortunately, it has been difficult to measure the annual temporal performance of private equity for portfolio models; we do not focus on these securities. The recommendations apply to fully integrated risk management systems (i.e. asset and liability management) with suitable extensions [5].

Alternative asset categories have become increasingly popular with institutional and wealthy individual investors since the recession in 2000-2001. The trend has been caused by several interrelated factors, including the superior performance achieved by leading university endowments over the past decade, and the need to recover lost surpluses by pension trusts (among others). Top university endowments in the U.S. (e. g. Yale, Harvard, Princeton, and Stanford Universities) and other leading institutional investors

1

have achieved 15-20[+] % annual returns over the past decade by shifting a large proportion of their capital to private investments. In contrast, especially since 2000, numerous pension trusts have fallen behind, with many funding ratios dropping to the 75-80% range [6].

A major benefit of the alternative investments involves the generation of return patterns that differ from factors that affect equity and bond markets. In particular, stocks and bonds are largely driven by three generic factors: 1) government (default free) interest rates; 2) corporate earnings as a proxy for the level of economic activities; and 3) a risk premium [7, 8]. Thus, an investor's diversification is limited due to the dependence on a relatively small number of underlying driving factors. And diversification will become much less during periods of economic instability and contagion – due to an increase in risk premium.

A second potential benefit of alternative assets, especially for private markets such as venture capital, is the ability to increase leverage while smoothing price variations over several years. By their structure, some private market securities, e.g., early stage ventures, are not subject to the fluctuations of liquid market based instruments. Due to the lack of reporting reliable returns on a regular basis, there is difficulty in analyzing these asset categories within an optimal portfolio model. Future research should be aimed at this domain (section 5). Accordingly, we focus on non-traditional assets possessing marketable securities in this paper.

For simplicity, we discuss the role of alternative investments for asset-only allocation models. To properly address an investor's circumstance, we advocate a comprehensive asset and liability model (ALM) such as, among others, described in Consigli and Dempster [9 and 10], Mulvey et al. [11] and Ziemba and Mulvey [5].

2. Multi-Period Portfolio Models

This section provides a brief explanation on benefits of adopting multi-period models, especially fixed mix policy rules, for portfolio construction. There are distinct advantages of a multi-period horizon, as compared with a static buy-and-hold framework [12 and 13]. First, the multi-period model can address a number of significant real world issues, such as transaction costs (e.g. taxes) and changing economic environments (growth versus recession) with non-constant correlation and covariance matrices. For instance, stock and bond returns are generally positively related under normal economic conditions, whereas these returns can become negatively related during and after a recession. A multi-period portfolio model can show the impact of these changing conditions on the investor's future wealth, in an integrated risk fashion. Also, the performance of a multi-period model can be greater than the performance of a buy-and-hold model for comparable planning horizons due to the gains attained by re-balancing the portfolio at selected time junctures. References [13, 14, 15, and 16] discuss the nature of the re-balancing gains.

There is a major drawback, however, to implement a multi-period model: the model can be non-convex, which makes it difficult to attain the optimal strategy. Instead of complicated optimization techniques, we provide a simple, yet efficient approach – fixed mix policy rule – to illustrate benefits of multi-period horizon. The fixed mix strategy always applies the same weights at the beginning of the time period to constituents, in contrast to the buy-and-hold approach, where the weights vary as the prices of constituents change over time. Also, the fixed mix strategy can serve as a benchmark for other dynamic strategies.

Early on, Samuelson [17] and Merton [18] showed that the fixed-mix investment rule is optimal under certain restrictive assumptions. Mulvey et al. [19], among others, presents a clear illustration for the connection between the fixed-mix rule and re-balancing gain. For simplicity, let's assume that there is one stock and one risk-free asset. Suppose the stock price process P_t follows a geometric Brownian motion that can be represented by the equation

$$dP_t = \alpha P_t dt + \sigma P_t dz_t,$$

where α is drift, σ is volatility and z_t is a Brownian motion with mean 0 and variance t. Similarly, risk-free asset B_t follows the same process with drift equal to 0 and volatility equal to 0. Then the stochastic differential equation for B_t can be written as

$$dB_t = rB_t dt.$$

Now, assume that we invest η in the stock and $(1-\eta)$ in the risk-free asset with the fixed mix policy rule. Then the wealth process of the portfolio W_t can be expressed as

$$\frac{dW_t}{W_t} = \frac{\eta dP_t}{P_t} + \frac{(1-\eta)dB_t}{B_t}.$$

After substituting the equation for P_t and B_t, one can show that the growth rate of the portfolio is

$$\gamma_w = \eta\alpha + (1-\eta)r - \frac{\eta^2\sigma^2}{2}.$$

For simplicity, we assume that the growth rate of the stock and the risk-free asset are equal.[1] Then the growth rate of the portfolio can be rewritten as

---

[1] This assumption is *not* required to illustrate the re-balancing gain, but it makes the illustration simpler and easier to understand.

$$\gamma_w = r + \frac{(\eta - \eta^2)\sigma^2}{2}.$$

If $0 < \eta < 1$, this quantity is greater than the growth rate of the buy-and-hold approach by $(\eta - \eta^2)\sigma^2 / 2$. The quantity corresponds to the re-balancing gain due to applying the fixed mix policy rule as compared with buy-and hold.

Many investors have applied versions of the fixed-mix rules with practical successes [5, 11, 16, 20, 21, and 22]. For example, the famous 60/40 norm (60% equity and 40% bonds) falls under this policy. Here, at each period, we rebalance the portfolio to 60% equity and 40% bond. Another good example is S&P 500 equal-weighted index (S&P EWI) by Rydex Investments. As opposed to traditional cap-weighted S&P index, stocks have the same weight (1/500) and the index is rebalanced semi-annually to maintain the weights over time. To illustrate the benefits of applying the fixed mix policy rule, during 1994~2005, S&P EWI achieved 2% excess return with only 0.6% extra volatility compared to S&P 500 index. Figure 1 illustrates log-prices of S&P 500 and S&P EWI for last 4 years.

Figure 1
Log-Prices of S&P 500 Index and S&P EWI during Jul.2003~Dec.2006
This figure illustrates the log price processes for S&P EWI and S&P 500 from Jul.2003 to Dec.2006. Each index is scaled to have a log-price of 0 at the beginning of the sample period. In term of the total return, S&P EWI outperformed S&P 500 index for last 4 years and this performance difference between 2 assets can be interpreted as a rebalancing gain due to the fixed mix policy rule.
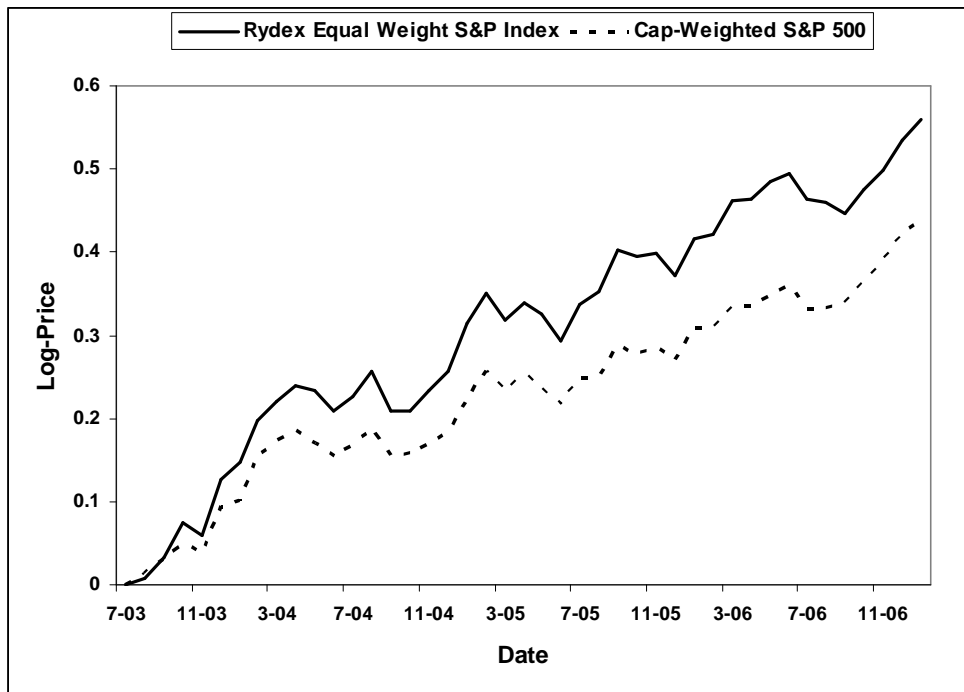
We close this section by discussing desirable properties of assets in order to achieve rebalancing gain. First, suppose two assets in the derivation above are perfectly correlated. Then, it can be easily shown that the rebalancing gain is zero. From this, it is evident that diversification among assets plays a major role to achieve an excess growth rate. This observation suggests that dynamic diversification is essential in order to produce extra gains via multi-period approaches. Also, as always, diversification provides a source of reducing portfolio risk. Second, given a set of independent assets, the rebalancing gain ($(\eta - \eta^2)\sigma^2/2$) increases as the volatilities of assets increase. To benefit from rebalancing gain, the volatility of each asset should be reasonably high. In this context, the traditional Sharpe Ratio might not be a good measure for individual asset in terms of multi-period portfolio management, even though it is still valid at the portfolio level. Additionally, low transaction costs (fees, taxes, etc.) are desirable, since applying the fixed mix policy rule requires portfolio rebalancing. In summary, the properties of the best ingredients for the fixed mix rule are: 1) relatively good performance (positive expected return); 2) relatively low correlations among assets; 3) reasonably high volatility; and 4) low transaction costs. For the more detailed discussion, see Fernholz [14] and Mulvey et al. [19].

3. Historical Perspective

In this section, we describe the performance of several major categories of alternative investments, along with traditional assets over a recent twelve-year historical period. While there are obvious limitations of evaluating historical performance, there is benefit for observing the past patterns. Figure 2 and Table 1 depict historical performance of some of the major traditional asset categories and alternative assets including an aggregate hedge fund index, managed futures index, a long/short equity fund index and a currency based index.

There are several general observations. First, the historical record is limited for most alternative assets. For instance, before 1994, the hedge fund industry was quite small and the indices did not adjust for survivor bias. In the future, due to the explosive growth in hedge funds, returns may decrease by reducing the attainable edge for certain strategies, such as statistical arbitrage. Given these limitations, the overall performance of the alternative assets was generally contained within the coverage of traditional equities, bonds, and real-estate investment trusts (REITs). The returns and risks are roughly compatible between traditional and alternative indices.

Several studies have conducted performance attribution of the return patterns of selected alternative assets, mostly hedge funds [2, 23, and 24]. This research helps investors understand the process undertaken by their portfolio managers (at least at a high level). If an investor can find a pattern that is a reasonably consistent match, the portfolio manager could be compensated by comparison to this benchmark, perhaps, in addition to the usual absolute return benchmarks. Also, the development of economic scenario generating systems requires a linkage of economic factors to the returns of the asset categories (both traditional and alternative) [10].

Figure 2
Performance of Alternative and Non-Alternative Asset Categories (1994~2005)

Among 12 categories, 4 assets – Tremont hedge fund aggregate index (Hedge Fund Ind.), Tremont long/short equity index (Tremont L/S), Currency Index and Tremont managed futures index (Man. Fut. Index) – are classified as alternative asset classes. Unlike traditional assets, each fund in these categories has a specific benchmark. That is, the money manager of a specific fund is asked to outperform the corresponding benchmark such as S&P500 or Rusell1000, while constructing her portfolio similar to the benchmark. Thus, performance of such alternative assets is highly dependent on their underlying benchmark. Therefore, a direct comparison between traditional and alternative assets is not straight forward. However, since we focus on the alternative assets as a genuine source of diversification rather than superior performance, we illustrate the historical performance of such assets along with traditional ones, in order to give a general idea to the readers. See footnote 1 for the detailed explanation of each asset.
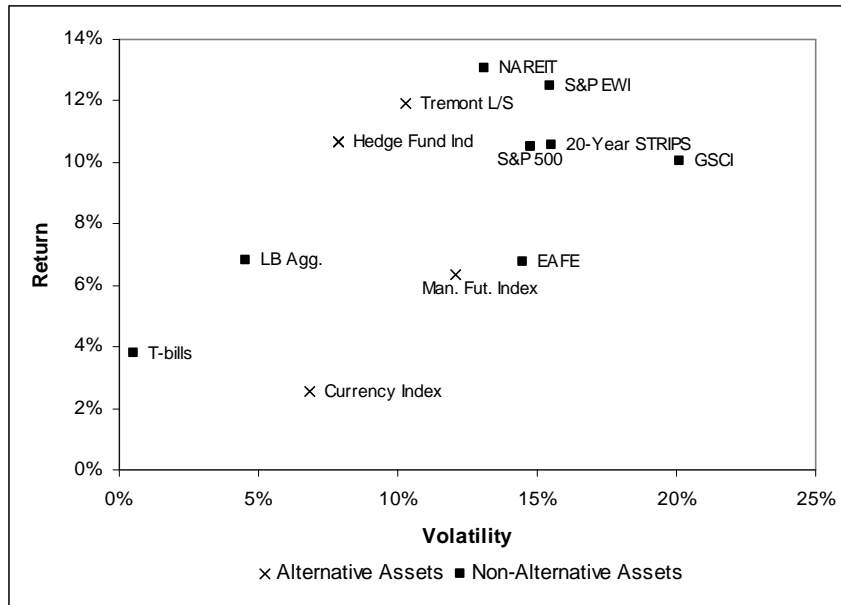
Traditional Asset Classes		
Type	**Name**	**Description**
Equity	S&P 500	Standard and Poor 500 index: An unmanaged cap-weighted index of 500 domestic stocks.
	S&P EWI	Rydex S&P 500 equal weighted index: The fixed mix version (equal weight) of S&P 500 index.
	EAFE	Morgan Stanley equity index for Europe, Australia, and the Far East: : An unmanaged cap-weighted index of overseas stocks.
Bond	LB Agg.	Lehman long aggregate bond index: An unmanaged index of government & corporate bonds, mortgage-backed securities and asset-backed securities.
	Strips	20-year U.S. government zero coupon bonds.
	T-bill	U.S. government 30-day Treasury bill
Real Estate	NAREIT	National Association of real estate investment trusts: An unmanaged index of U.S. real estates.
Commodity	GSCI	Goldman Sachs commodity index: A composite index of long-only commodity futures
Alternative Asset Classes		
Type	**Name**	**Description**
Hedge Fund	Hedge Fund Ind	Tremont hedge fund aggregate index: An asset-weighted hedge fund index which is net of fees and expenses.
	Man. Fut. Index	Tremont managed futures index: An asset-weighted hedge fund index of investments in listed bond, currency, equity and commodity futures markets.
	Tremont L/S	Tremont long/short equity index: An asset-weight hedge fund index of investments on both the long and short sides of equity markets.
Currency	Currency Index	Reuters-CRB Currencies Index: An index of 5 currency futures (BP, EC, CD, SF and JY).

6

Table 1
Summary Statistics for Historical Performance of Popular Asset Categories

In this table, investment performance of each asset category for whole sample period (top), the first 6-year (middle) and the second 6-year (bottom) is shown. Assets with relatively high maximum drawdown are highlighted. For detailed description of each asset, see the legend in Figure 1.

Whole Sample Period (1994~2005)

	Annualized Return	Standard Deviation	Sharpe Ratio	Maximum Drawdown	Return/Drawdown
S&P 500	10.5%	14.8%	0.45	44.7%	0.24
LB Agg.	6.8%	4.5%	0.66	5.3%	1.29
EAFE	6.8%	14.5%	0.20	47.5%	0.14
T-bills	3.8%	0.5%	0.00	0.0%	N/A
NAREIT	13.1%	13.1%	0.71	26.3%	0.50
GSCI	10.1%	20.1%	0.31	48.3%	0.21
Hedge Fund Ind	10.7%	7.9%	0.87	13.8%	0.77
Man. Fut. Index	6.4%	12.1%	0.21	17.7%	0.36
Currency Index	2.6%	6.8%	-0.18	28.7%	0.09
Tremont L/S	11.9%	10.3%	0.78	15.0%	0.79
S&P EWI	12.5%	15.4%	0.56	30.3%	0.41
20-Year STRIPS	10.6%	15.5%	0.43	22.8%	0.46

First Sub Period (1994~1999)

	Annualized Return	Standard Deviation	Sharpe Ratio	Maximum Drawdown	Return/Drawdown
S&P 500	23.5%	13.6%	1.37	15.4%	1.53
LB Agg.	5.9%	4.0%	0.24	5.2%	1.14
EAFE	12.3%	13.8%	0.54	15.0%	0.82
T-bills	4.9%	0.2%	0.00	0.0%	N/A
NAREIT	6.5%	12.0%	0.13	26.3%	0.25
GSCI	4.7%	17.4%	-0.01	48.3%	0.10
Hedge Fund Ind	14.1%	9.9%	0.93	13.8%	1.02
Man. Fut. Index	5.5%	11.5%	0.05	17.7%	0.31
Currency Index	0.1%	6.7%	-0.73	20.4%	0.00
Tremont L/S	18.5%	11.6%	1.18	11.4%	1.62
S&P EWI	17.1%	13.7%	0.89	19.9%	0.86
20-Year STRIPS	7.2%	14.7%	0.16	22.8%	0.32

Second Sub Period (2000~2005)

	Annualized Return	Standard Deviation	Sharpe Ratio	Maximum Drawdown	Return/Drawdown
S&P 500	-1.1%	15.2%	-0.25	44.7%	-0.03
LB Agg.	7.7%	5.1%	0.99	5.3%	1.47
EAFE	1.5%	15.1%	-0.08	47.5%	0.03
T-bills	2.7%	0.5%	0.00	0.0%	N/A
NAREIT	20.0%	13.9%	1.24	15.3%	1.31
GSCI	15.7%	22.5%	0.57	35.4%	0.44
Hedge Fund Ind	7.4%	5.1%	0.92	7.7%	0.96
Man. Fut. Index	7.2%	12.7%	0.35	13.9%	0.52
Currency Index	5.1%	7.0%	0.34	15.3%	0.33
Tremont L/S	5.6%	8.6%	0.34	15.0%	0.37
S&P EWI	8.1%	17.0%	0.31	30.3%	0.27
20-Year STRIPS	14.0%	16.3%	0.69	19.1%	0.73

Next, we review briefly the historical performance of the asset categories over two distinct sub periods. We designate the first period January 1, 1994 to December 31, 1999 as "high equity", where as the second period January 1, 2000 to December 31, 2005 indicates "low equity". Over the entire twelve year period, the annual returns for the asset categories range from low = 2.6% (for currencies) to high = 13.1% (for real estate investment trusts - REITs). Many assets display disparate behavior over the two six-year sub periods: The Goldman Sachs commodity index (GSCI) and REITs had their worst showing during the first sub period – the lowest returns and highest draw-down values, whereas EAFE and S&P500 display the opposite results – high returns in the first sub period. As a general observation, investors should be ready to encounter sharp drops in individual asset categories. Draw-down for half of the categories lies in the range 26% to 48% (Tables 1).

Two of the highest historical return-to-risk ratios occurred in the hedge fund categories: 1) the Tremont aggregate hedge fund index (.87); and 2) the Tremont long/short index (.78). In both cases, returns are greater than the S&P 500 index with much lower volatility. As mentioned, this performance has led to increasingly interest in hedge funds. Many experts believe that the median future returns for hedge funds are likely to be lower than historical values – due in part to the large number of managers entering the domain. In fact, low volatility may be a detriment for increasing overall portfolio performance since it limits the rebalancing gains. There are advantages to combining assets with modest return-to-risk ratios and reasonable returns in a rebalanced portfolio, when the lower ratio is caused by higher volatility.

In summary, alternative assets have displayed solid performance over the twelve year period, 1994~2005, especially the Tremont aggregate hedge fund and long/short indices. In both cases, however, the returns in the second period, while remaining positive, fell substantially, partially due to the lower returns of equities. In contrast, the currency index and managed futures showed the opposite relationship – higher returns in the second period. The later assets showed countercyclical behavior as compared with equities. As mentioned, there is some concern that returns will drop further with the recent expansion of the alterative investment universe. Even given this environment, alternative investments can provide benefits to the investor, as a novel source of diversification, as we will see in the next section.

4. The Role of Alternative Assets in the Portfolio Management

As mentioned, there is evidence that private markets can generate superior returns as compared with many public markets [25]. Unfortunately, for most investors, top opportunities are rarely available without special access privileges. These accessibility issues are slowly receding with the recent introduction of tradable hedge funds and related instruments (such as active exchange traded funds), which allow individual investors to gain a portion of the median hedge-fund returns.

Table 2
Portfolio Description

Portfolio	Description	Constituents
P1	Traditional Assets Only (Buy-and-Hold)	Traditional assets: SP500, LB Bond, EAFE, NAREIT, GSCI, STRIPS
P2	With Alternative Assets (Buy-and-Hold)	Traditional assets: SP500, LB Bond, EAFE, NAREIT, GSCI, STRIPS Alternative assets: Man Futures, Hedge Fund Ind., L/S Ind., Currency
P3	With Alternative Assets (Fixed Mix)	Traditional assets: *SP EWI*, LB Bond, EAFE, NAREIT, GSCI, STRIPS Alternative assets: Man Futures, Hedge Fund Ind., L/S Ind., Currency

Importantly, alternative investments can provide the benefits of wide diversification and leverage to achieve superior performance. In this section, we are less concerned with superior performance; we employ the alternative assets to provide additional sources of diversification – above and beyond that dictated by equities and bonds.

The most comprehensive approach for evaluation of alternative assets in a portfolio is to apply an integrated risk management system on a set of investment vehicles which includes alternative asset classes. However, such an approach is beyond the scope of this article. Thus, rather than conducting an ALM optimization, we apply the fixed mix rule to the assets mentioned in the previous section. More specifically, the analysis will construct three portfolios: (P1) a buy-and-hold portfolio of only traditional assets, (P2) a buy-and-hold portfolio of traditional and alternative assets, (P3) a fixed mix portfolio of both traditional and alternative assets. In this regard, we employ the fixed-mix rule at two levels. First at the stock selection level, we substitute an equal-weighted S&P 500 index for the capital-weighted S&P 500 fund. The equal weighted index has generated better performance over the standard S&P500 index [21], as would be expected due to the additional returns gotten from re-balancing the mix. Then, the portfolio is rebalanced monthly to fulfill the fixed mix policy rule at the asset selection level. For simplicity, assets are weighted equally for all three portfolios. Table 2 summarizes these strategies.

We first compare P1 and P2 to illustrate the diversification benefits from the alternative asset categories. The two leftmost columns of Table 3 show the resulting performance. Here, the historical performance of P1 and P2 is 9.9% and 9.4% per year, with annualized volatility equal to 7.9% and 6.6%, respectably. As expected, alternative assets serve as a novel source of diversification, resulting P2 have higher risk-reward ratios. The benefit of including alternative assets becomes even greater when the fixed mix rule is employed. Among three portfolios, P3 shows the best performance in most of performance measures. Clearly, wide diversification pays off in terms of reducing the portfolio's overall risk – volatility and maximal drawdown. The maximum drawdown for P3 is mere 6.4%, which is almost half of P1's. Also, improvements in return-risk ratios are significant, especially the return-drawdown ratio (from 0.89 to 1.54). It is also worth noting that the performance of P3 in each of the two sub periods (1994-1999, and 2000-2005) is relatively similar from one another, which implies that it provides more reliable outcomes.

Table 3
Historical Results with Different Leverage Values Applied to Portfolios

In this table, historical investment performance of 3 portfolios is illustrated for the whole sample period (1994~2005, top row), the first 6-year sub-period (1994~1999, middle row) and the second sub-period (2000~2005, bottom row). Performance of P1, P2 and P3 are shown in the left, middle and right column, respectably. As anticipated, P3 outperforms both P1 and P2, which depict the benefits of including alternative assets as well as adopting multi-period models. Performance improvements are most significant in the return-drawdown ratio. Also, the fixed mix portfolio with alternative assets (P3) shows the best return-risk ratios as it gets levered up.

		P1: Traditional Assets Only (Buy-and-Hold)				P2: With Alternative Assets (Buy-and-Hold)				P3: With Alternative Assets (Fixed Mix)			
	Leverage	0%	20%	50%	100%	0%	20%	50%	100%	0%	20%	50%	100%
1994 ~ 2005	Return	9.9%	11.0%	12.8%	15.5%	9.4%	10.4%	12.1%	14.7%	9.8%	10.9%	12.7%	15.6%
	Volatility	7.9%	9.5%	11.9%	15.9%	6.6%	7.9%	9.9%	13.2%	6.2%	7.4%	9.3%	12.4%
	Sharpe Ratio	0.76	0.76	0.75	0.74	0.84	0.84	0.83	0.82	0.96	0.96	0.95	0.95
	Drawdown(DD)	11.1%	14.1%	18.6%	25.7%	7.1%	9.3%	12.7%	18.3%	6.4%	8.0%	10.4%	14.4%
	Return/DD	0.89	0.78	0.69	0.61	1.31	1.12	0.95	0.80	1.54	1.37	1.22	1.08
1994 ~ 1999	Return	11.1%	12.3%	14.0%	16.9%	10.9%	12.1%	13.9%	16.7%	9.6%	10.5%	11.9%	14.1%
	Volatility	7.9%	9.5%	11.8%	15.7%	6.8%	8.1%	10.1%	13.5%	6.3%	7.5%	9.4%	12.5%
	Sharpe Ratio	1.40	1.30	1.19	1.07	1.61	1.49	1.37	1.24	1.53	1.40	1.26	1.13
	Drawdown(DD)	9.1%	11.0%	14.0%	19.1%	7.1%	8.7%	11.0%	14.9%	6.4%	8.0%	10.4%	14.4%
	Return/DD	1.22	1.11	1.00	0.88	1.53	1.39	1.25	1.12	1.51	1.32	1.14	0.98
2000 ~ 2005	Return	10.9%	12.5%	15.0%	19.0%	9.2%	10.5%	12.4%	15.6%	9.9%	11.3%	13.5%	17.1%
	Volatility	7.9%	9.5%	11.9%	15.8%	6.5%	7.8%	9.7%	13.0%	6.2%	7.4%	9.3%	12.4%
	Sharpe Ratio	1.38	1.32	1.26	1.20	1.42	1.35	1.27	1.20	1.61	1.53	1.45	1.38
	Drawdown(DD)	7.3%	9.4%	12.6%	17.8%	5.3%	7.0%	9.6%	13.9%	4.7%	6.3%	8.7%	12.6%
	Return/DD	1.50	1.33	1.19	1.07	1.75	1.49	1.29	1.12	2.10	1.81	1.56	1.35

Figure 3
Efficient Frontiers of the Portfolios with/without Alternative Assets

Left figure illustrates efficient frontiers in volatility-return plane, while right one is drawn in maximum drawdown-return plane. The efficient frontier of P3 contains those of P1 and P2 in both cases, which clearly exhibit the role of alternative assets in portfolio construction.
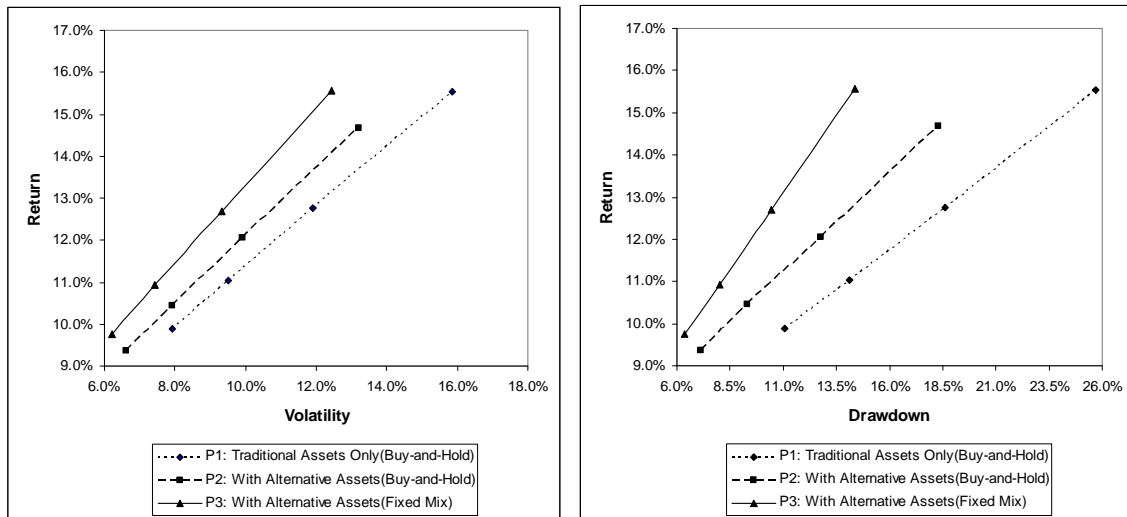
Next, we take three portfolios and apply selected degrees of leverage – at several values: 20%, 50%, and 100%. Leverage is achieved in the conventional way – by borrowing money at the T-bill rate and putting it on the constituents accordingly. Note that since three portfolios in consideration do not include t-bill, the relative weights do not change as the portfolio is levered. The returns increase for each juncture, with increasing risks (as measured by volatility and drawdown). However, the fixed mix portfolio with alternative assets (P3), the overall risks are quite reasonable even at the 100% leverage – 12.4% annualized volatility and 14.4% drawdown, resulting better risk-reward ratio than the other portfolios. Interestingly, P3 outperforms P1 with no leverage in terms of return-risk ratios, even at 100% leverage. Efficient frontiers in Figure 3 clearly illustrate this point. See Mulvey [19] for further improvements via overlay strategies.

The historical results suggest that investors can benefit by including alternative assets in their portfolio. First, an investor with access to the top deals can achieve truly superior performance – such as Renaissance Technologies' annual return equal to over 35% after fees since 1989. Similarly, the leading U.S. university endowments have shown that private investments can be highly profitable. But also significantly, alternative assets offer the benefits of combing wide diversification and targeted leverage. These advantages are more readily available for most investors than gaining access to the top private investments.

There are two qualifiers for this empirical study: 1) the historical performance of alternative investments may not correspond to future performance due to, among others, the increase in the number of hedge funds existing today; and 2) it can be difficult to rebalance a portfolio due to restriction on the entry and exit of capital within many of the private markets. Accordingly, the empirical results should be treated as an illustration of possible benefits. This issue is expected to be partially resolved in a near future due to emergence of new financial instruments such as active exchange traded funds. The main message remains – alternative investments can provide increasing diversification benefits due to the uniqueness of the return patterns.

5. Summary and Future Directions

The top alternative investments have delivered superior performance over the past 10-15 years, as shown by the returns of leading university endowments and the consistently high returns of selected hedge funds. Unfortunately, most investors are unable to gain access to these opportunities at this time.

The report suggests that, with careful risk management, investor performance can be improved by adding alternative assets to a portfolio of traditional asset categories. Alternative assets can provide reasonable performance with less dependency on the usual economic factors such as corporate earnings, interest rates, and risk premium. The novel return patterns provide a substantial benefit for increasing diversification. For long-term investors, wide diversification can be coupled with target leverage to increase portfolio performance. Rebalancing gains are also available for selected investors. As always,

investors should carefully analyze their potential risks and rewards in an integrated, anticipatory fashion.

What are directions for future research? First, we can continue to search for assets with novel sources of returns (as compared with stocks, bonds, and money market securities). A prime example involves weather-related products. Ideally, the emerging securities would develop in liquid markets so that investor has valid market prices and can achieve rebalancing gains.

In addition, research can be aimed at improving the modeling/pricing of private securities. Current approaches, such as the internal rate of returns for seasoned (vintage-year) ventures, are not so helpful for the problem of seeking an optimal asset allocation. Approaches developed for asset allocation (and integrated risk management) under traditional categories will need to be extended for the inclusion of their privately held investments/securities.

Undoubtedly, long term, multi period financial planning models for individual investors will continue to grow in popularity. The aging population of wealthy individuals will require assistance as they approach retirement and for estate planning purposes. The U.S. government has recently passed legislation which makes it easier for financial organizations to provide probabilistic investment advice. This change in regulation has already led to implementation of a number of stochastic planning systems (similar to the ones discussed in this paper). Individual and institutional investors alike can benefit by applying integrated risk management systems in conjunction with a full set of traditional and alternative asset categories.

References

1. Anson MJP. Handbook of Alternative Assets, 2nd Edition; John Wiley: Hoboken, NJ, 2006.

2. Feng W, Hsieh DA. A primer on hedge funds. Journal of Empirical Finance 1999 6: 309-311.

3. Jaeger RA. All about Hedge Funds; McGraw-Hill: 2003.

4. Lhabitant F-S. Hedge Funds, Myths and Limits; John Wiley and Sons: 2002.

5. Ziemba W, Mulvey J.(eds.) World Wide Asset and Liability Modeling; Cambridge University Press: Cambridge, UK, 1998.

6. Mulvey JM, Simsek KD, Zhang Z, Fabozzi F, Pauling W. Assisting defined-benefit pension plans. Operations Research (to appear) 2006.

7. Bakshi G, Chen Z. Stock valuation in dynamic economies. Journal of Financial Markets 2005 8 issue 2: 111-151.

8. Chen Z, Dong M. Stock valuation and investment strategies. Yale ICF Working Paper No. 00-46 2001.

9. Consigli G, Dempster MAH. Dynamic stochastic programming for asset-liability management. Annals of Operations Research 1998 81: 131-161.

10. Consigli G, Dempster M. The CALM stochastic programming model for dynamic asset-liability management. World Wide Asset and Liability Modeling (Ziemba W, Mulvey J, eds); Cambridge University Press: Cambridge, UK, 1998; pp 464-500.

11. Mulvey JM, Gould G, Morgan C. An asset and liability management system for Towers Perrin-Tillinghast. Interfaces 2000 30: 96-114.

12. Mulvey JM, Pauling B, Madey RE. Advantages of multiperiod portfolio models. Journal of Portfolio Management 2003a 29: 35–45.

13. Luenberger D. Investment Science; Oxford University Press: New York, 1997.

14. Fernholz R. Stochastic Portfolio Theory; Springer-Verlag: New York, 2002.

15. Fernholz R, Shay B. Stochastic portfolio theory and stock market equilibrium. Journal of Finance 1982 37: 615-624.

16. Mulvey JM, Kaul SSN, Simsek KD. Evaluating a trend-following commodity index for multi-period asset allocation. Journal of Alternative Investments 2004 summer: 54-69.

17. Samuelson PA. Lifetime portfolio selection by dynamic stochastic programming. Review of Economics Statistics 1969 51: 239-246.

18. Merton RC. Lifetime portfolio selection under uncertainty: the continuous-time case. Review of Economics Statistics 1969 51: 247-257.

19. Mulvey JM, Ural C, Zhang Z. Improving performance for long-term investors: wide diversification, leverage, and overlay strategies. Quantitative Finance (to appear) 2007.

20. Mulvey JM, Thorlacius AE. The Towers Perrin global capital market scenario generation system: CAP Link. World Wide Asset and Liability Modeling (Ziemba W, Mulvey J, eds); Cambridge University Press: Cambridge, UK, 1998.

21. Mulvey JM, Essential portfolio theory. A Rydex Investment White Paper (also Princeton University report) 2005.

22. Perold AF, Sharpe WF. Dynamic strategies for asset allocation. Financial Analysts Journal 1988 1/2: 16–27.

23. Feng W, Hsieh DA. Performance characteristics of hedge funds, and commodity funds: natural versus spurious biases. Journal of Financial and Quantitative Analysis 2000 35: 291-307.

24. Feng W, Hsieh DA. Benchmarks of hedge funds performance: information contents and measurement biases. Financial Analyst Journal 2002 58: 22-34.

25. Swensen DF. Pioneering Portfolio Management; The Free Press: New York, 2000.